Proofs

Proofs

<u>Here are the types of proofs we need to be able to do…</u>

1) Proving “for all” statements
2) Showing a “for all statement” is false
3) Proving an “if-then” statement
4) Showing an “if-then’ statement is false
5) Mathematical Induction (another time)

Proofs

Some symbols

$\forall$ stands for "for all" , "for each" , or "for every"

$\exists$ stands for "there exists" , or "there is"

$\in$ stands for "in" , "is a member of" , or "is an element of"

Proofs

To prove a "for all" statement $\forall\ x \in G\ ,\ P(x)$

- In a "for all" statement, there is a group of objects
- The "for all" statement is claiming that some property is true for every object in the group
- Since the group of objects is usually infinite, you don't want to check that the property is true for each object in the group one at a time (because you will never finish)
- So you start by selecting an arbitrary member of the group without ever mentioning which specific object you selected
- Then you use any known facts about that object until you conclude that the object you selected has the property
- Then since you never disclosed which object from the group you chose and still concluded that the property is true, the property must be true for every object of the group

Proofs

Ex 1: Prove the following…

a) $\forall \; x \in [-1,3]\,,\;\; -7 \leq 5x - 2 \leq 13$

b) $\forall \; x \in [7,9]\,,\;\; 17 \leq 2x + 3 \leq 21$

Proofs

<u>To prove a “for all” statement with cases</u>

- In a “for all” statement, there is a group of objects
- The “for all” statement is claiming that some property is true for every object in the group
- Sometimes to prove the property for everything in the group, it’s easier to divide the group into smaller groups and prove the property is true for each smaller group one at a time
- Divide the work into cases
- Prove the property if the object you selected is from the first smaller group
- Proceed the same way you prove any “for all” statement
- Then do this for each of the smaller groups until you have finally proved the result for everything in your original group

Proofs

<u>Ex 2</u>: Prove the following…

a) $\forall \ x \in \mathbb{R}, \ x^2 \geq 0$

b) For all integers n, $n^2 + 3n + 1$ is odd

c) $\forall \ x \in [-1,1], \ x^2 \in [0,1]$

d) For all integers n, $n^2 + n$ is even

Proofs

Some things to know about integers

- The integers are closed under addition

  This means that if you take any 2 integers and add them, the result will also be an integer.
- The integers are closed under multiplication

  This means that if you take any 2 integers and multiply them, the result will also be an integer.

Def:

1. An integer n is even if there is another integer m such that $n = 2m$.
2. An integer n is odd if there is another integer m such that $n = 2m + 1$.

Proofs

<u>To disprove a “for all” statement</u>

- The negation of the “for all statement” $\forall\ x \in G,\ P(x)$
  is the “there exists statement” $\exists\ x \in G$ such that $\sim P(x)$
- To disprove a “for all statement”, you need to find a specific object in the group where the statement is not true

Proofs

Ex 3: Disprove the following…

a) $\forall \; n \in \mathbb{N}$, $2^n + 1$ is prime

b) $\forall \; n \in \mathbb{N}$, $n^2 + 3n + 2$ is a multiple of 3

Proofs

<u>To prove an “if-then” statement</u> If P then Q

- An “if-then” statement is almost the same as a “for all” statement
- Whatever comes after the word “if” (P) is trying to describe the group of objects the statement is about
- Whatever comes after the word “then” (Q) is the property that the statement is claiming is true for all of the objects
- To prove an “if-then” statement, start of by selecting an arbitrary object in the group. This time, you will select an object that makes P true
- Then prove that Q is true for that object
- Since you never specified what object you selected, the statement must be true for all objects in the group

Proofs

Ex 4: Prove the following…

a) $\forall \ x \in \mathbb{R}$, if $\sin(x) = 1$ then $\cos(x) = 0$

b) $\forall \ x \in \mathbb{R}$, if $x > 4$ then $3 - x < -1$

Proofs

To disprove an "if-then" statement   If P then Q

- Just like disproving a "for all" statement, you need to find a specific object in the group (an object that makes P true) that doesn't make the property (Q) true.

Proofs

Ex 5: Disprove the following…

a) $\forall \ x \in \mathbb{R}$, if $\sin(x) = 0$ then $\cos(x) = 1$

b) $\forall \ x \in \mathbb{R}$, if $x < 1$ then $x^2 < 1$